EE 232: Lightwave Devices
Lecture #11 – Gain in quantum wells

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2/25/2019

Transparency condition for quantum well (estimation)

$$n = \sum_{subbands} N_n = \sum_n \int_0^\infty \frac{\rho_{2D}}{L_z} f(E) dE$$

(total carrier density)

$$N_n = \int_0^\infty \frac{m_e^*}{\pi \hbar^2 L_z} \frac{dE}{1 + \exp[(E - F_c)/kT]}$$

(carrier density within single subband)

$$\text{Let } x = (E - F_c)/kT$$

$$N_n = \int_{(E_g + E_{en} - F_c)/kT}^{\infty} \frac{m_e^*}{\pi \hbar^2 L_z} \frac{dx}{1 + e^x}$$

$$= \frac{m_e^* kT}{\pi \hbar^2 L_z} [x - \ln(1 + e^x)] \Big|_{(E_g + E_{en} - F_c)/kT}^{\infty}$$

$$\boxed{N_n = \frac{m_e^* kT}{\pi \hbar^2 L_z} \ln\left(1 + \exp[(F_c - E_g - E_{en})/kT]\right)}$$

similarly,

$$\boxed{P_m = \frac{m_h^* kT}{\pi \hbar^2 L_z} \ln\left(1 + \exp[(E_{hm} - F_v)/kT]\right)}$$

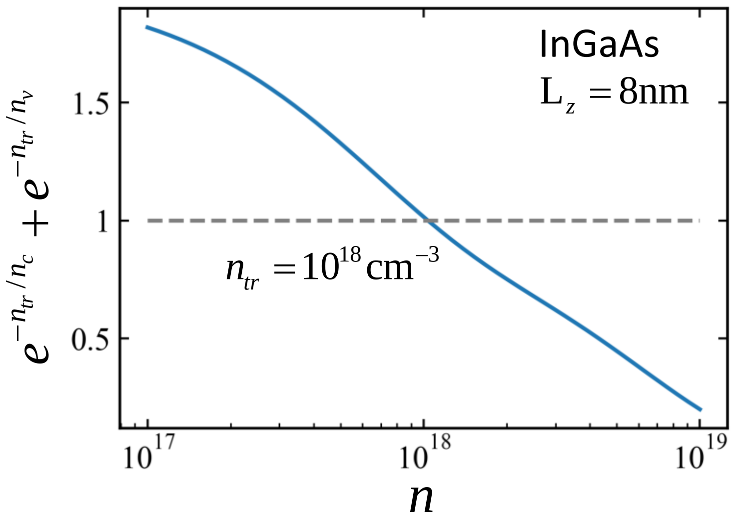
Transparency condition for quantum well (estimation)

If only **one** subband is filled,

$$F_c = kT \ln\left[\exp\left(n\frac{\pi\hbar^2 L_z}{m_e^* kT}\right) - 1\right] + E_g + E_{en}$$

$$F_v = -kT \ln\left[\exp\left(p\frac{\pi\hbar^2 L_z}{m_h^* kT}\right) - 1\right] + E_{hm}$$

$$F_c - F_v = E_{hm}^{en}$$ (transparency condition)



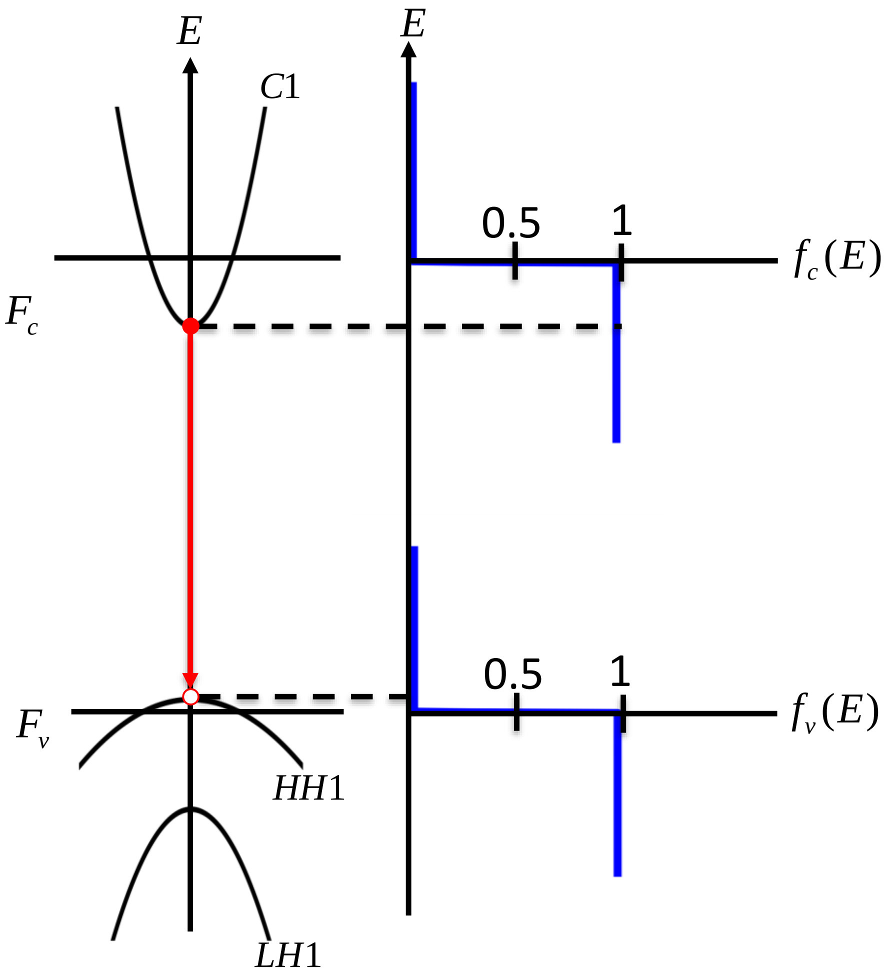
After several lines of algebra, and assuming quasi-neutrality (n=p), we obtain an equation in terms of transparency carrier density →

$$e^{-n_{tr}/n_c} + e^{-n_{tr}/n_v} = 1$$

where, $n_c = \dfrac{m_e^* kT}{\pi\hbar^2 L_z}$

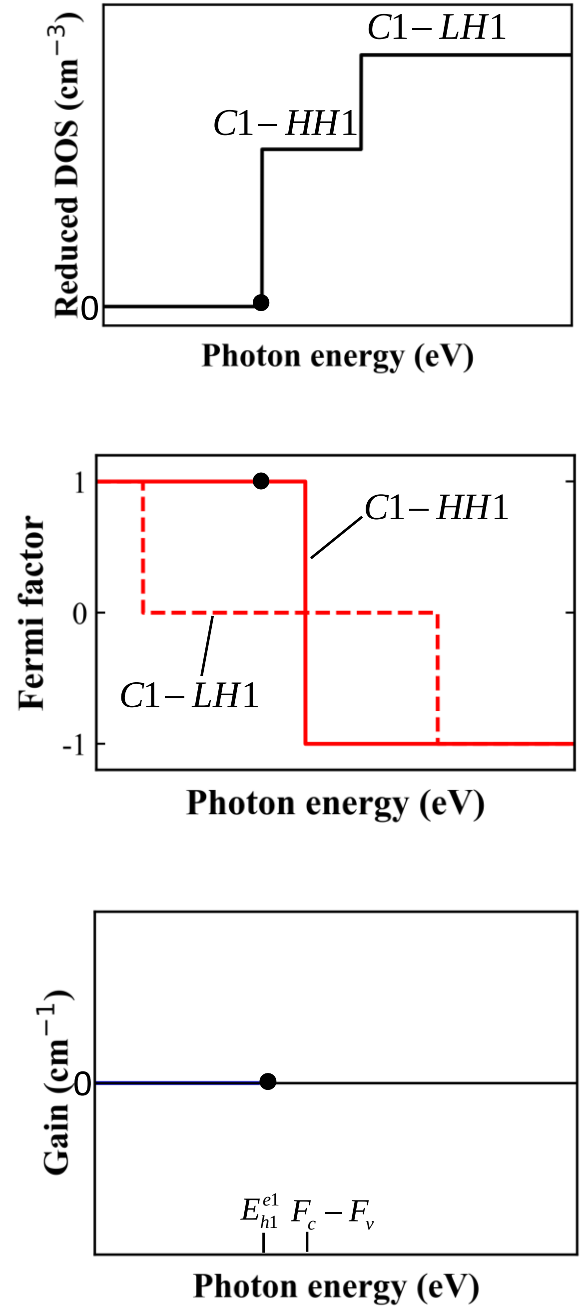
$$n_v = \frac{m_h^* kT}{\pi\hbar^2 L_z}$$

Gain spectrum (T=0K)

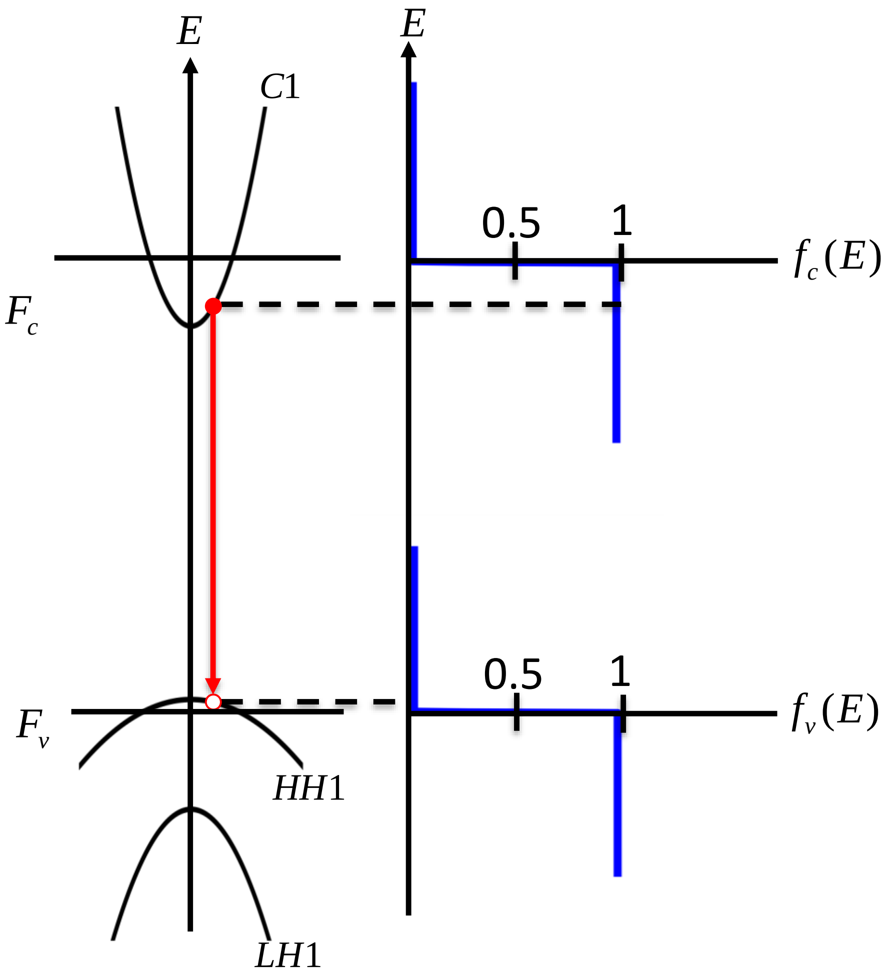


$$g(\hbar\omega) = -\alpha(\hbar\omega)$$
$$= g_p\left[f_c(\hbar\omega) - f_v(\hbar\omega)\right]$$

Fermi inversion factor



Gain spectrum (T=0K)



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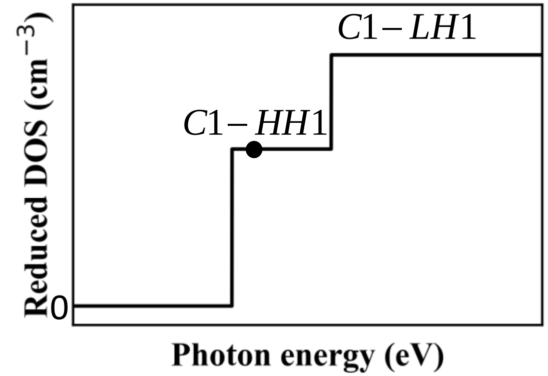
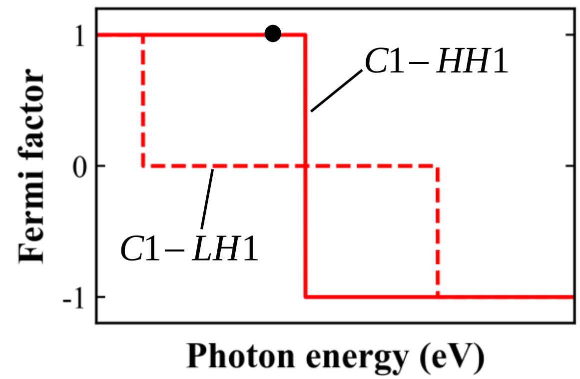
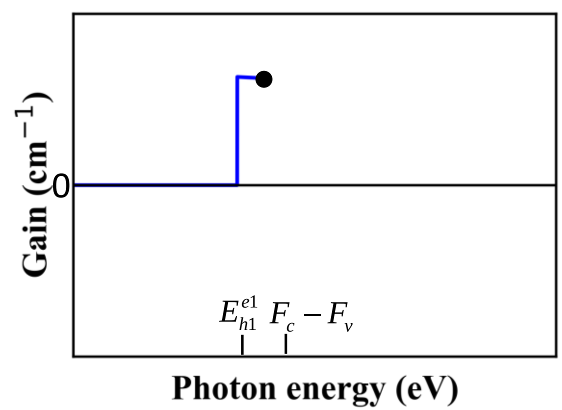
↑
Fermi inversion factor







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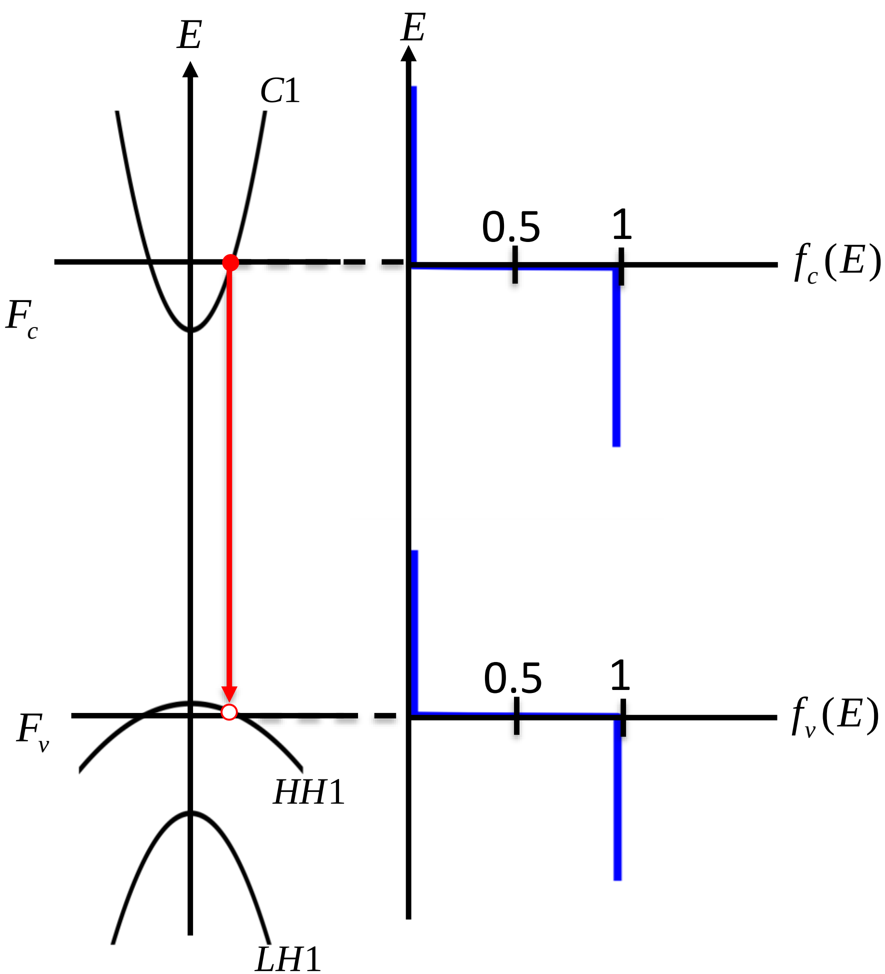

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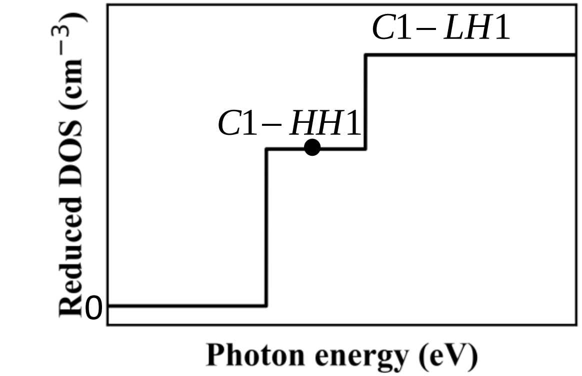
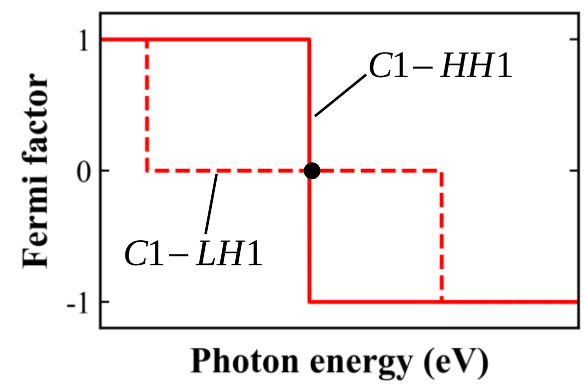
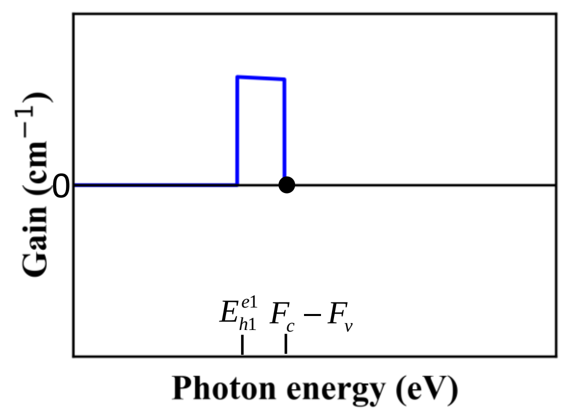
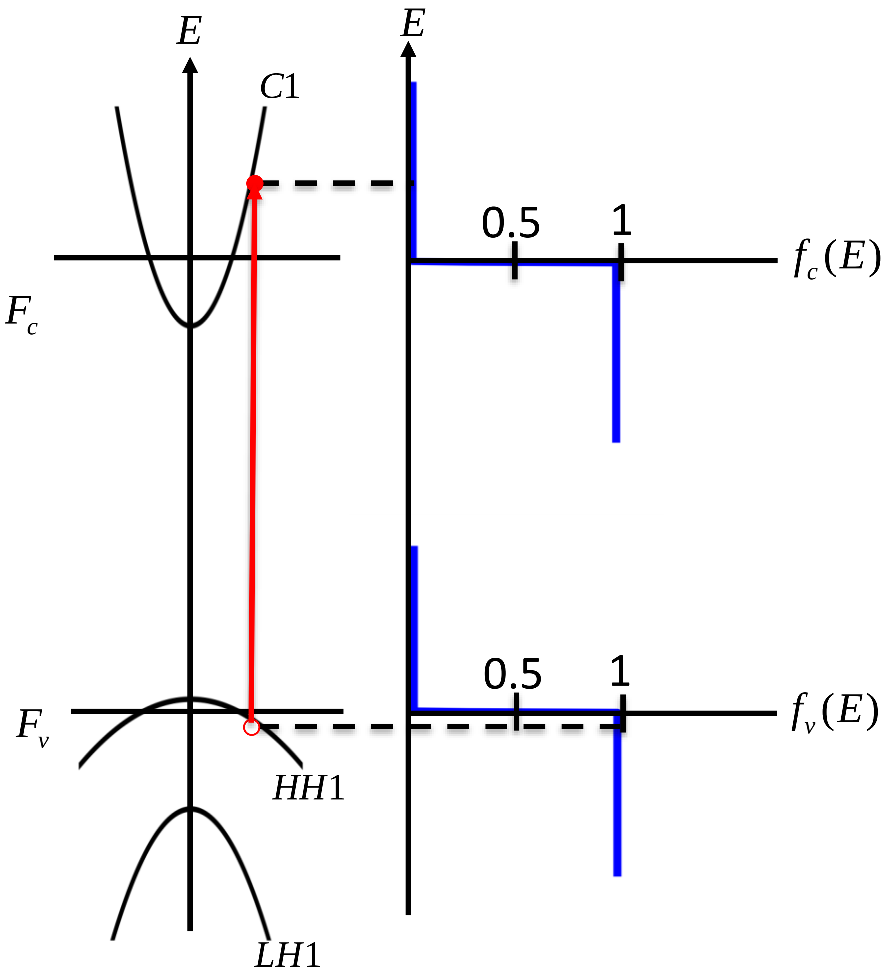
Fermi inversion factor







Gain spectrum (T=0K)



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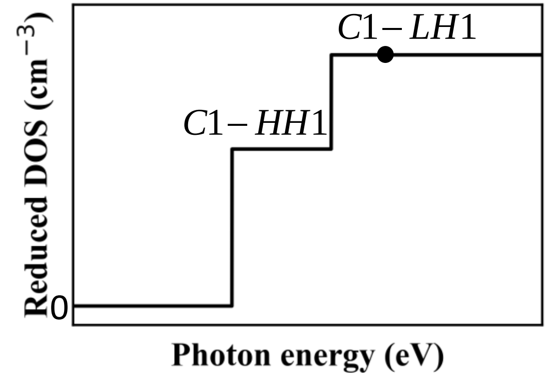
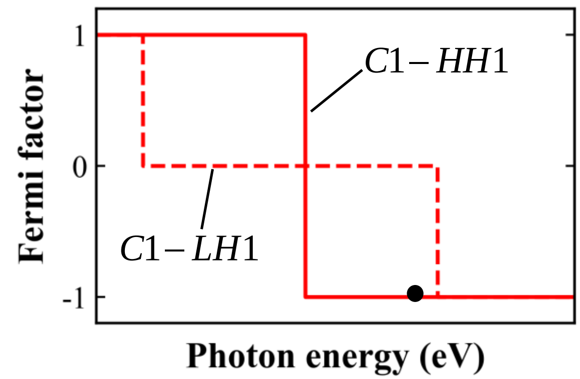
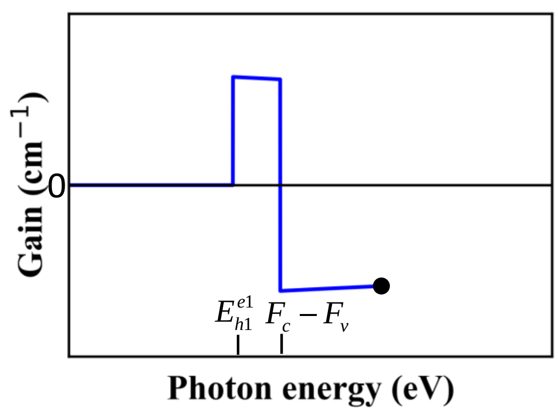
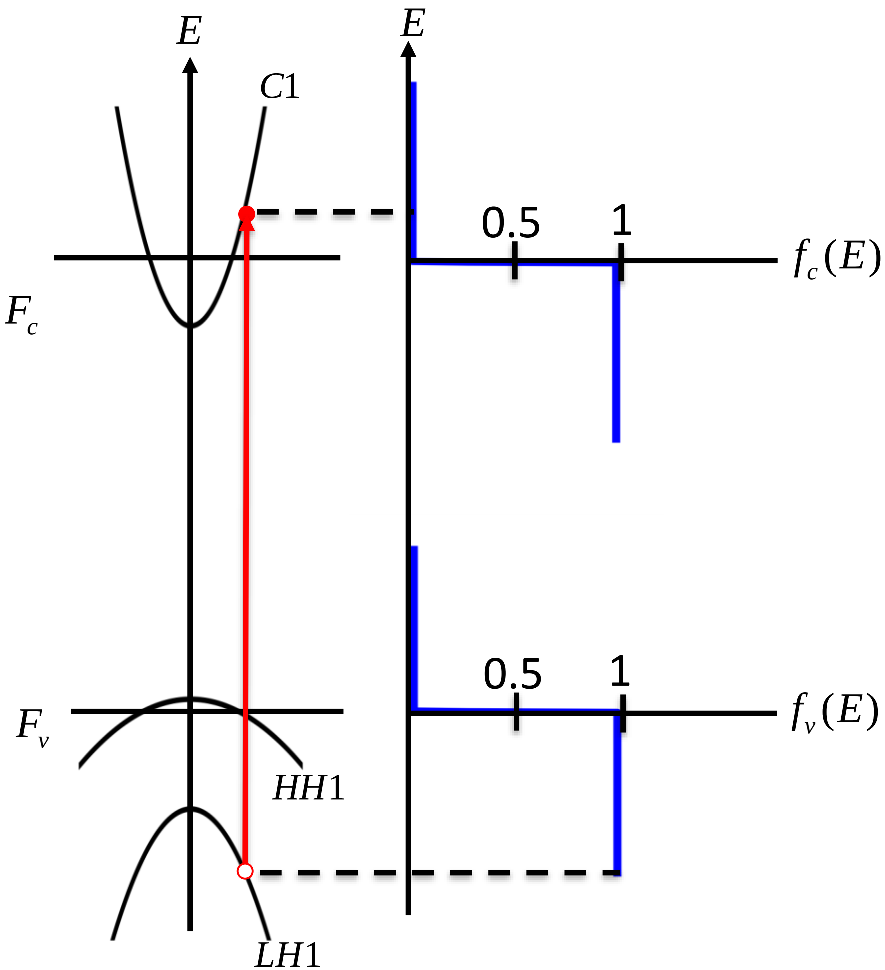
Fermi inversion factor







Gain spectrum (T=0K)



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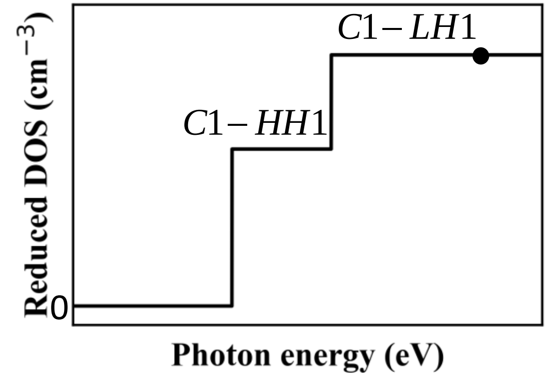
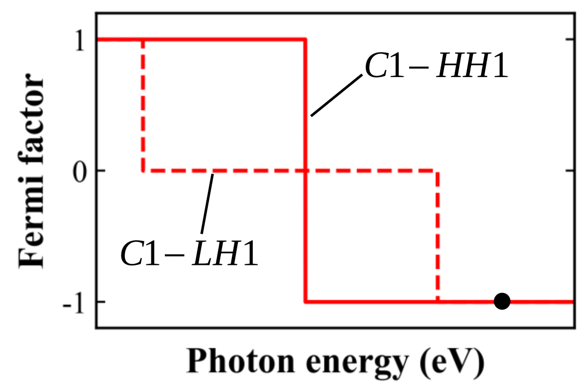
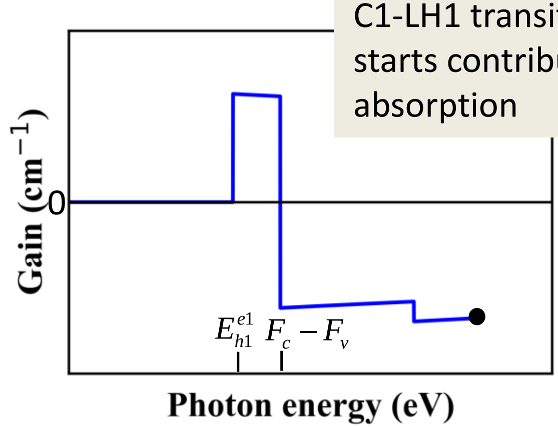
Fermi inversion factor

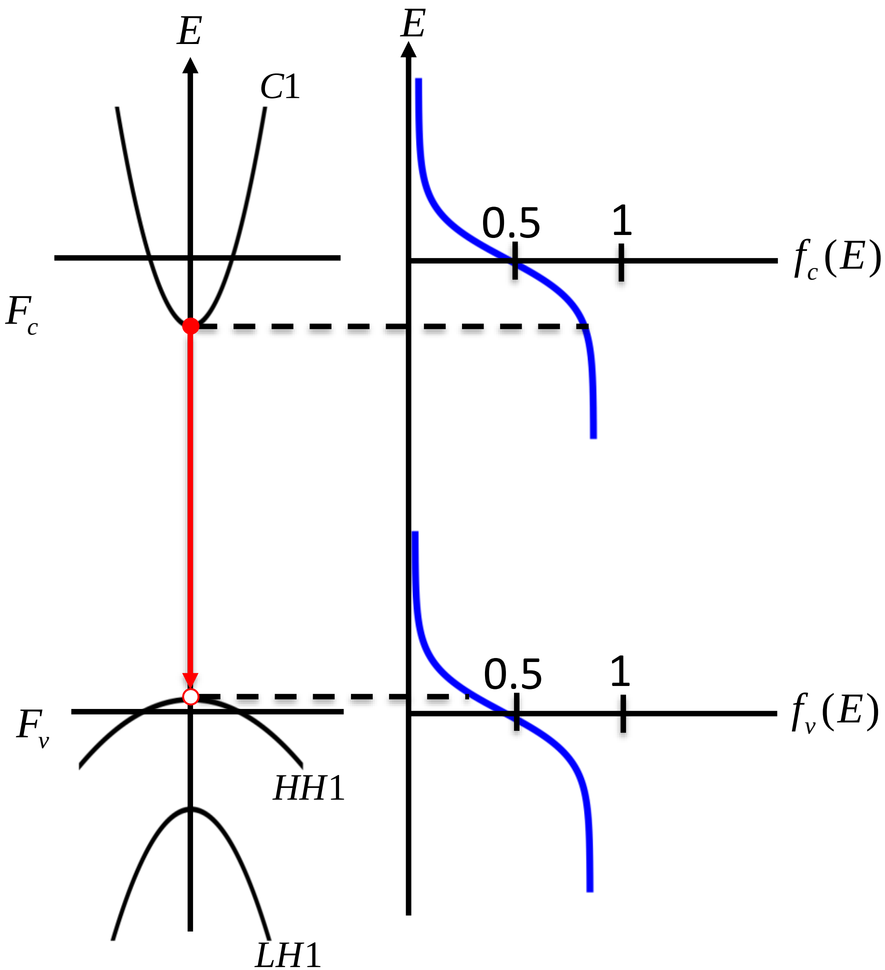




C1-LH1 transition starts contributing to absorption



Gain spectrum (T=300K)



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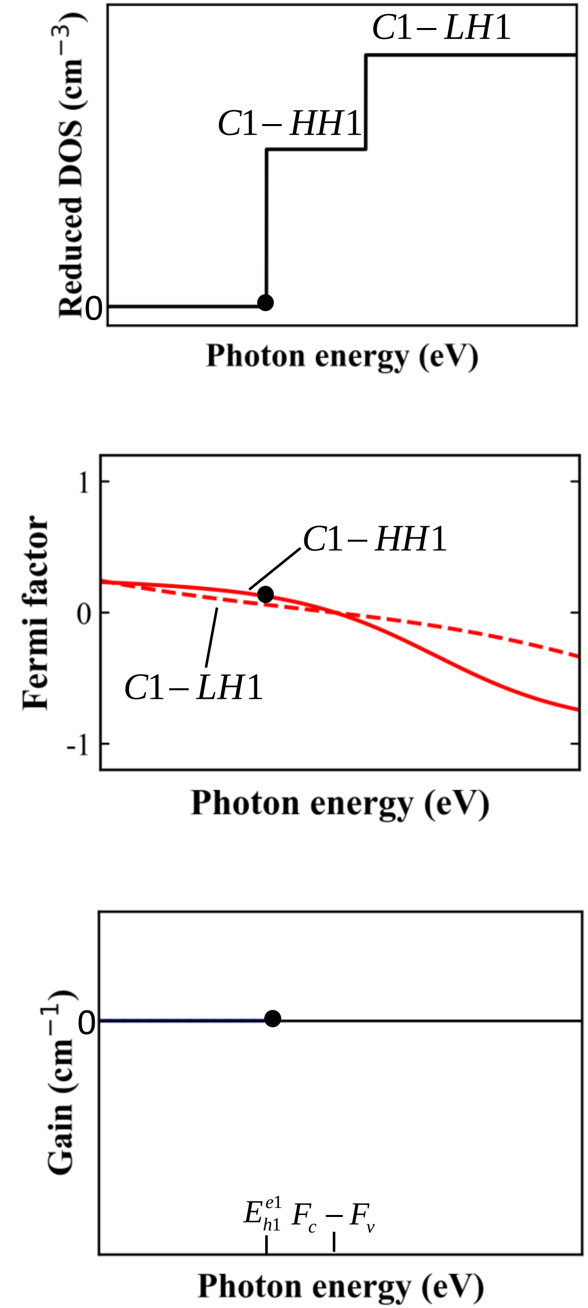

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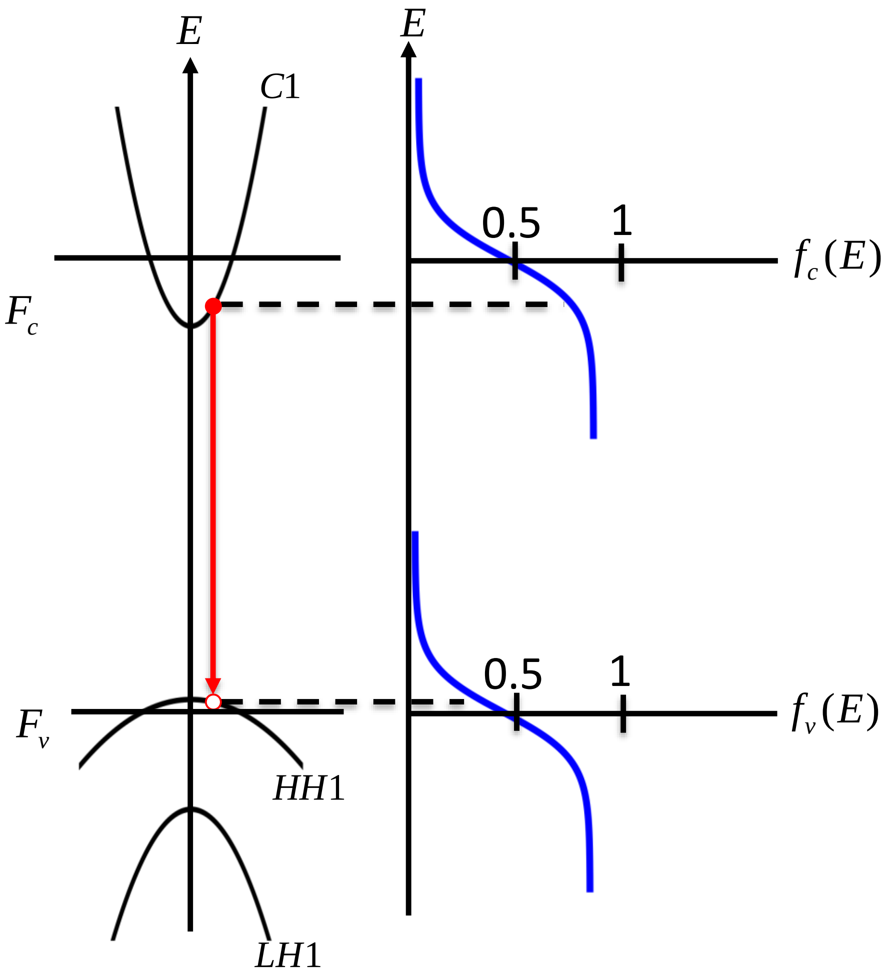

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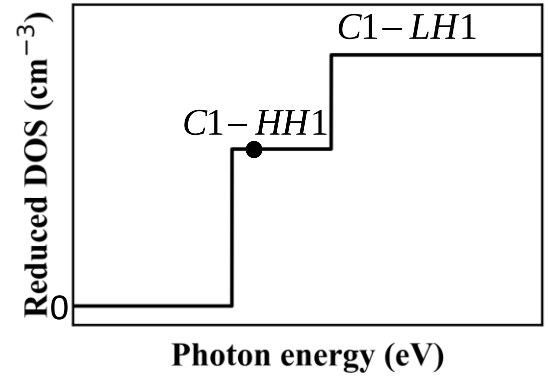
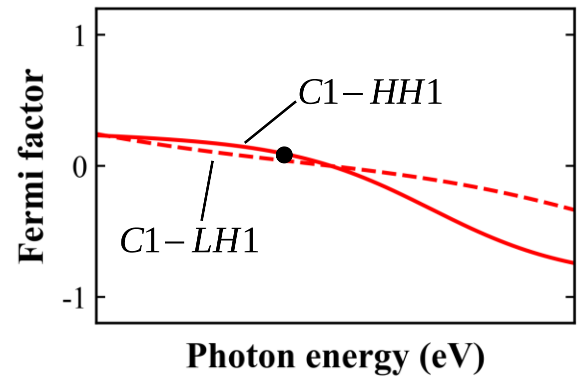
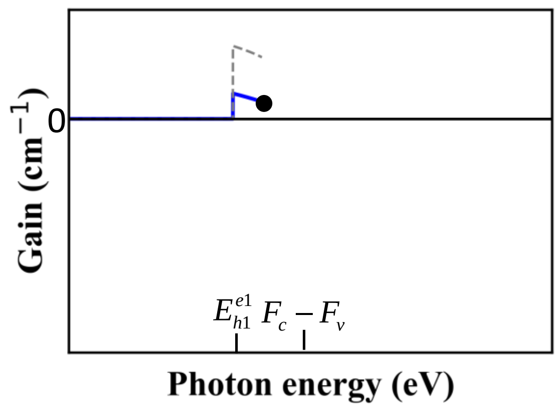
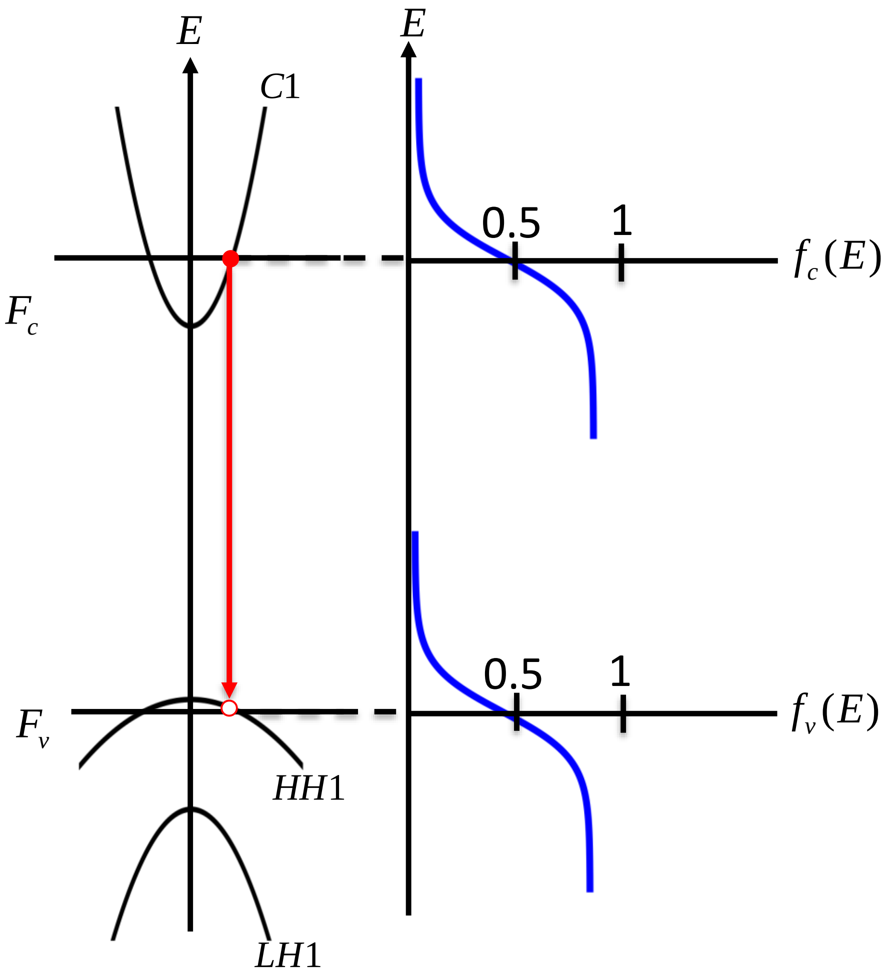






Gain spectrum (T=300K)



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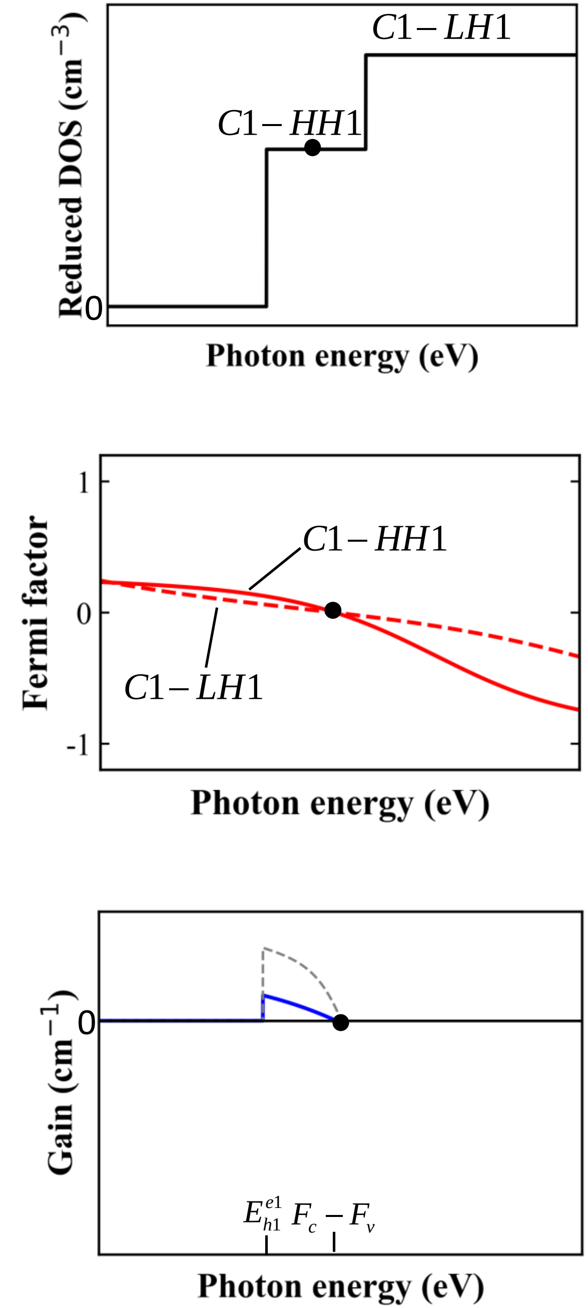
Fermi inversion factor



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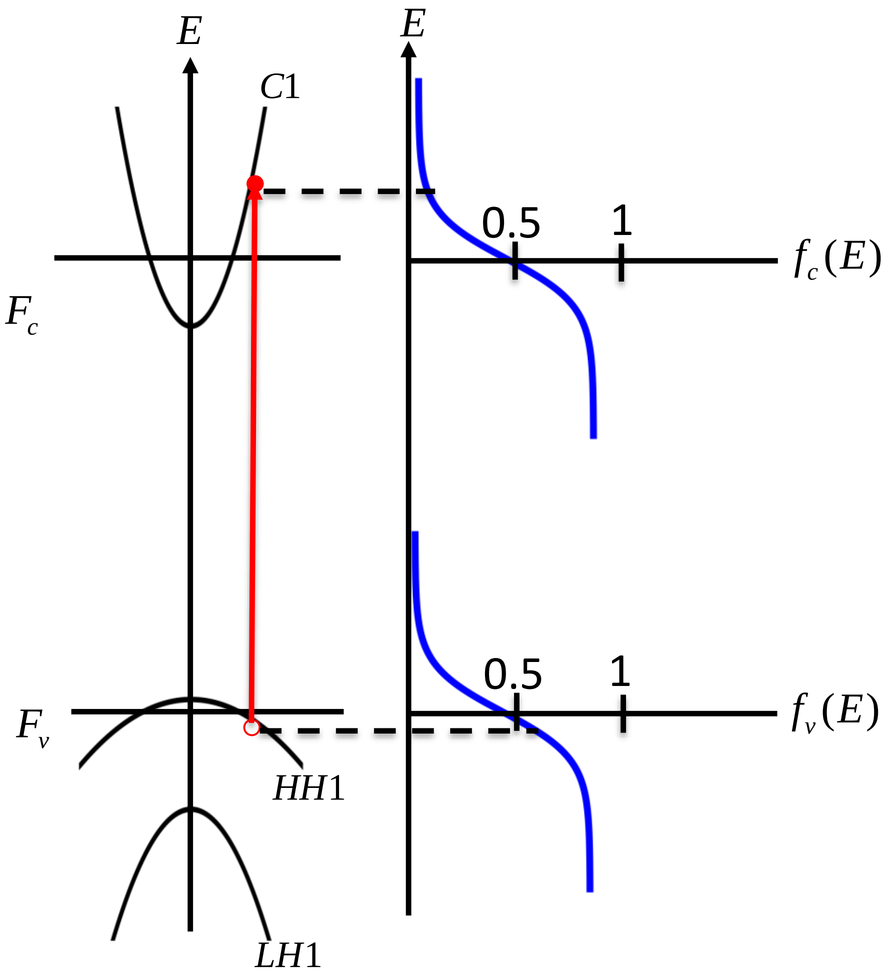

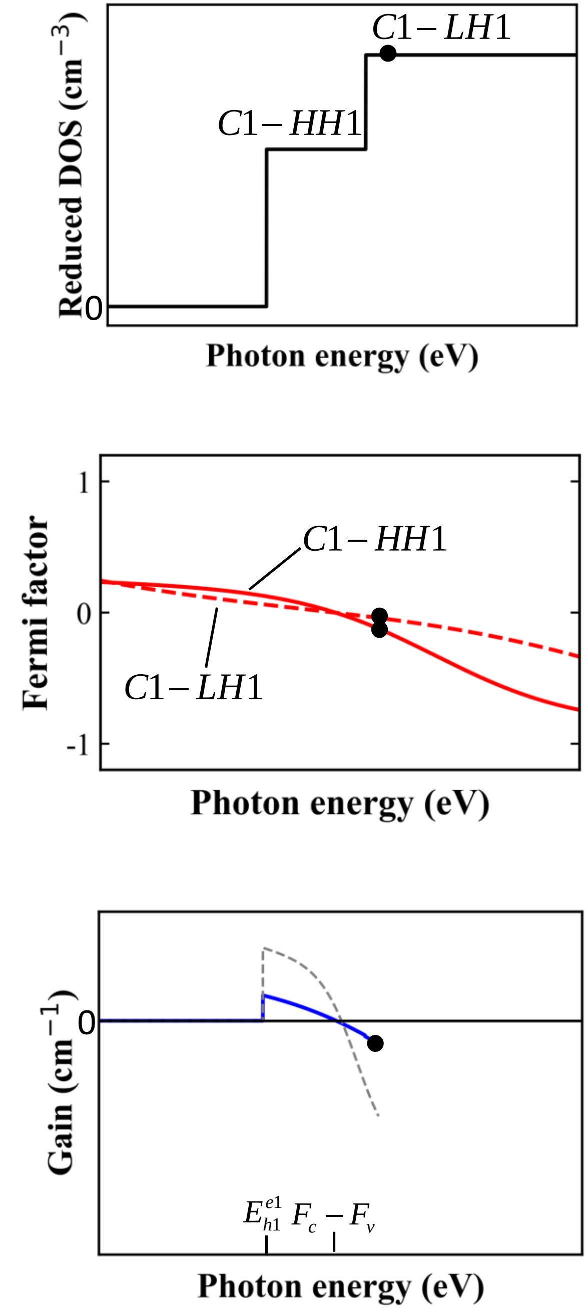

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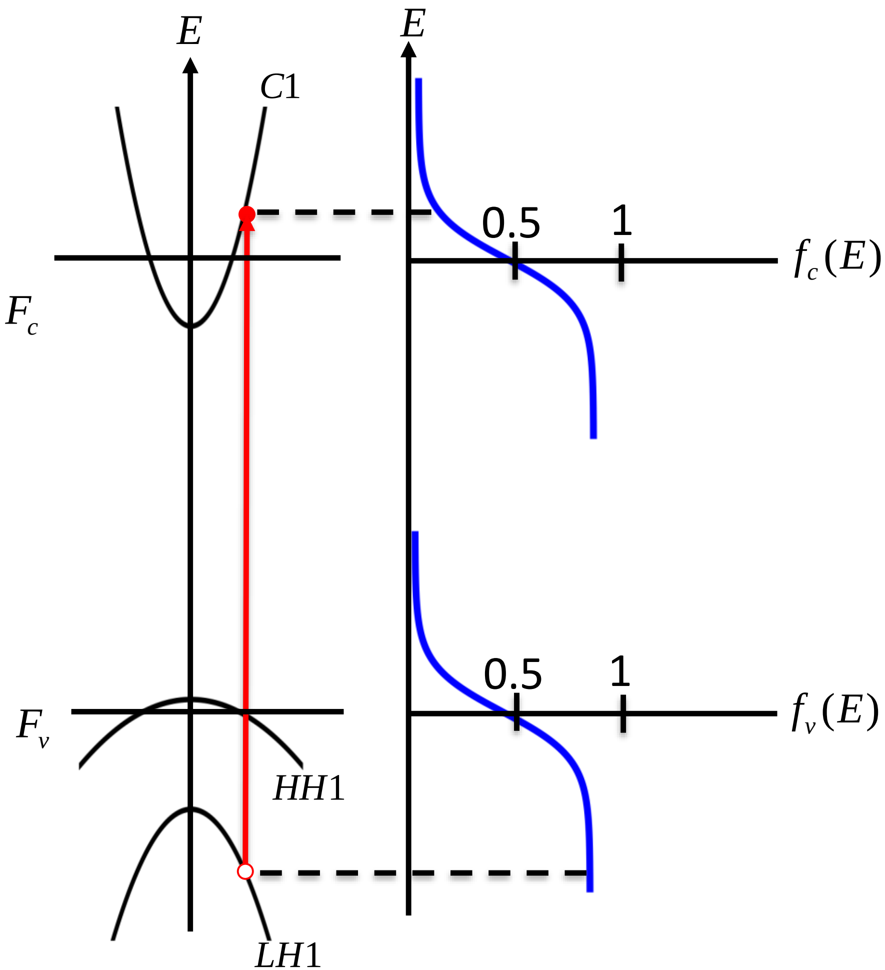

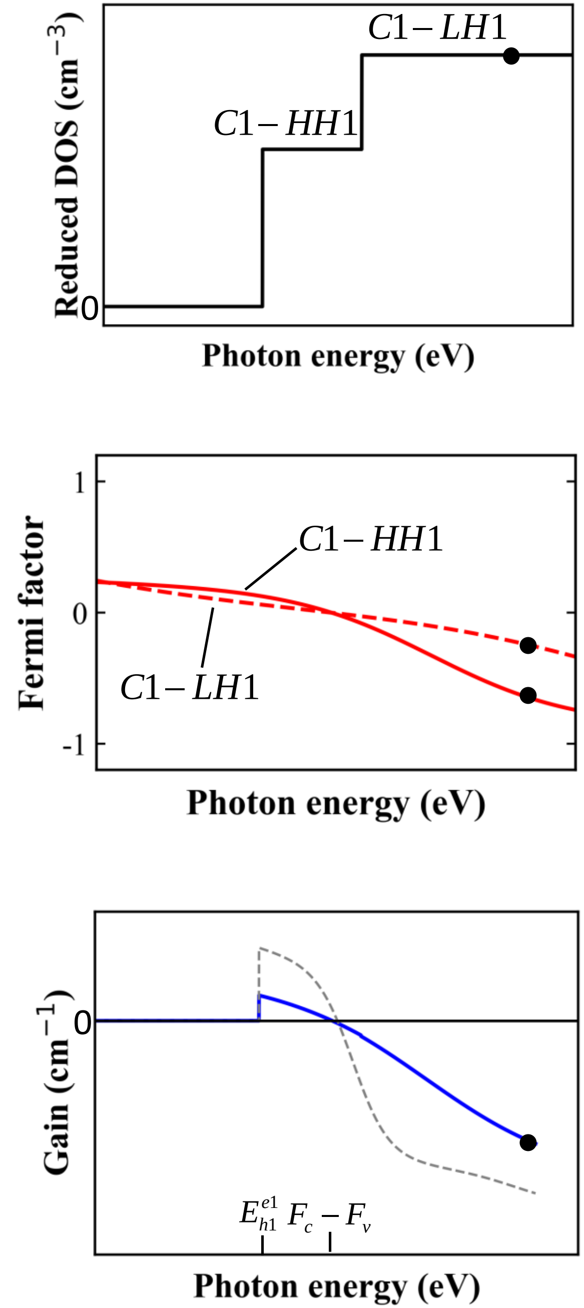

$(\hbar\omega)$

$$- g_p \left[f_c(\hbar\omega) - f_v(\hbar\omega) \right]$$

Fermi inversion factor

Gain spectrum

$InP/In_{0.53}Ga_{0.47}As$ quantum well
(Transverse electric mode)
$L_z = 6$ nm

$$g(\hbar\omega) = -\alpha(\hbar\omega)$$
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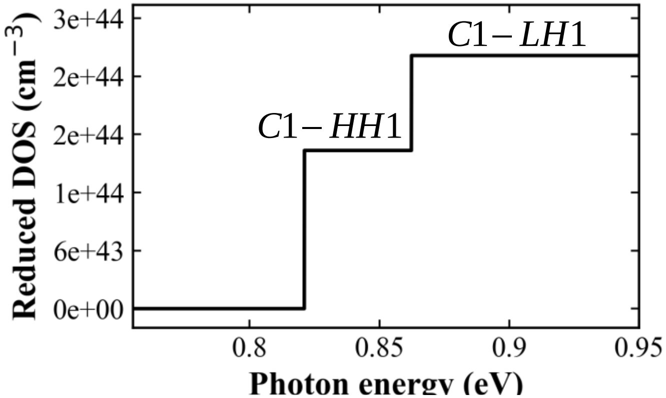
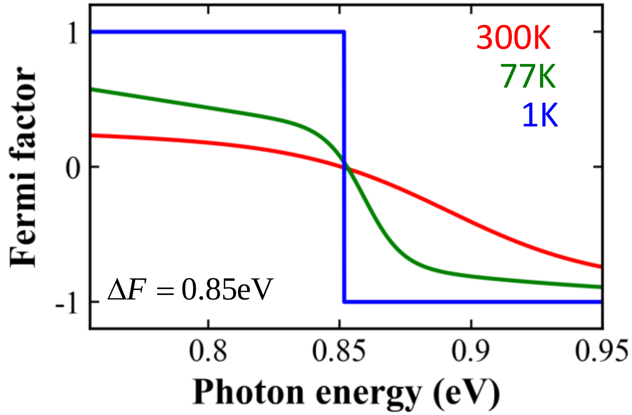
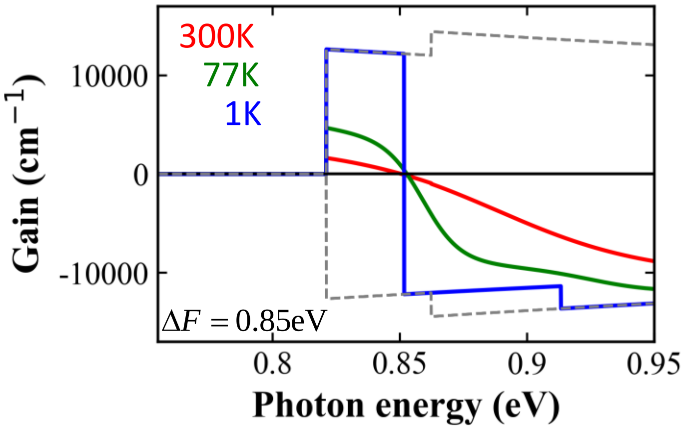
↑
Fermi inversion factor







Bandgap temperature dependence is ignored here

Gain spectrum

InP/$In_{0.53}Ga_{0.47}As$ quantum well
(Transverse electric mode)

$L_z = 6$ nm

$$g(\hbar\omega) = -\alpha(\hbar\omega)$$
$$= g_p\left[f_c(\hbar\omega) - f_v(\hbar\omega)\right]$$

↑ Fermi inversion factor